Design of Beams

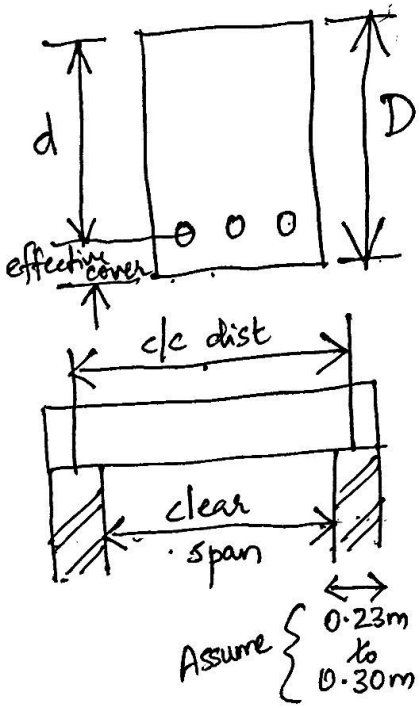
Singly reinforced beam

Given data:
1. Type of beam – SS; Cant
2. Span of beam – effective span, clear span
3. Loads on beam – UDL (kN/m)
4. Grade of concrete, steel (M20, M25) (Fe415, Fe500)

Steps involved in design

1. find effective depth (d)

Effective span = Min of { clear span + d ✗ ; c/c dist b/w supports ✓ }



$\frac{\text{Effective span}}{\text{Effective depth}} \leq 7$ (for cantilevers)

$\frac{\text{Effective span}}{\text{Effective depth}} \leq 20$ (for SSB)

$\Rightarrow$ from this effective depth (d) $\geq$ _____ mm

Assume effective cover = 50mm

$\therefore$ Overall depth D = d + effective cover = _____ mm

2. Find width of beam (from fig 1, Pg 34)

Min. width $b = 200\,mm$

3. Compute moment to be resisted by beam

case 1: If designed to carry a load w kN/m, then w includes DL & LL.

case 2: If beam is subjected to external load or imposed load W_L kN/m, then self weight W_D kN/m should be added to W_L kN/m to get total load.

- Once total load is computed, factor it.

Ultimate load or factored load (W_u) $= 1.5 \times W$ kN/m

- Finally, compute factored moment or Ultimate moment

$$M_u = \left(\frac{W_u l^2}{8}\right) \leftarrow \text{SSB}$$

UDL

For cantilevers $M_u = \frac{W_u l^2}{2}$

$l \rightarrow$ effective span

4. find limiting moment of resistance of beam

$$M_{ulim} = 0.36 f_{ck}\, b\, x_{umax} (d - 0.42\, x_{umax})$$

5. Compare M_u & M_{ulim}

- if $M_u < M_{ulim} \Rightarrow$ Design as singly reinforced beam
- if $M_u > M_{ulim} \Rightarrow$ Design as Doubly reinforced beam

6. Find the type of section. $x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$

from $M_u = 0.36 f_{ck} b x_u (d - 0.42 x_u)$,

find $x_u =$ _____ mm.

Compare x_u with x_{umax}

- if $x_u < x_{umax} \Rightarrow$ Under-reinforced section
  ∴ OK
- if $x_u > x_{umax} \Rightarrow$ Over-reinforced section
  ∴ Redesign.

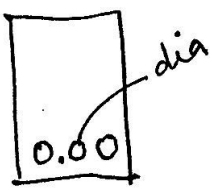
7. Find Ast

Use $x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$

$\Rightarrow A_{st} =$ _____ mm^2

from code, $A_{st,min} = \frac{0.85 bd}{f_y}$ (Pg 47)

$A_{st,max} = 0.04 bD$



∴ Provide A_{st} = _____ mm^2

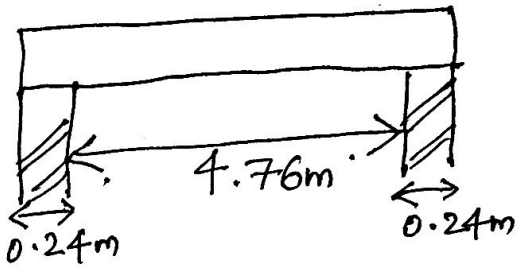
Using ϕ mm dia bars,

No: of bars reqd = $\frac{A_{st}}{\pi/4 \phi^2} \approx$ ___ bars

8mm
10mm
12mm
16mm
20mm
25mm
32mm
36mm
40mm

(Q.) Design a simply supported beam of clearspan 4.76m to carry 16 kN/m uniformly distributed load. Use M20 concrete & Fe415 steel.

Ans:



1. Find d

effective span = c/c dist b/w supports

Assume support width = 0.24m

$\Rightarrow$ effective span l = 4.76 + 0.24

= 5.00m

= 5000 mm

for SSB, $\frac{l}{d} \leq 20$

$\frac{5000}{d} \leq 20 \Rightarrow d \geq \frac{5000}{20}$

≥ 250 mm

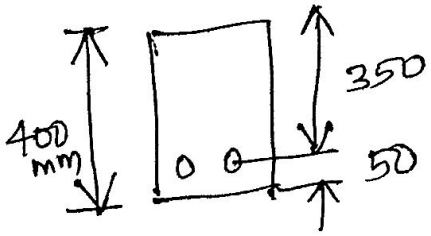
Take d = 350 mm

Assume effective cover = 50 mm

$\therefore$ Overall depth D = d + effective cover

= 350 + 50

= 400 mm



Effective span l = min. { clear span + d, or c/c dist b/w supports }

= min { 4.76 + 0.35 = 5.10 m, 5.00 m }

= 5.00 m ✓

2. Width of beam = min. 200 mm

(b) = 240 mm (assume)

3. $w = 16\ kN/m$

factored load (W_u) = $1.5\,w = 1.5 \times 16$

$= 24\ kN/m$

factored moment (M_u) = $\frac{W_u l^2}{8} = \frac{24 \times 5.00^2}{8}$

$= 75\ kNm$

4. $M_{ulim} = 0.36\, f_{ck}\, b\, x_{umax} (d - 0.42\, x_{umax})$

$= 0.36 \times 20 \times 240 \times (0.48 \times 350) \times (350 - 0.42 \times 0.48 \times 350)$

$= 81.12 \times 10^6\ Nmm$

$= 81.12\ kNm$

$f_{ck} = 20\ N/mm^2$

$b = 240\ mm$

$x_{umax} = 0.48d$ (for Fe 415)

$= 0.48 \times 350$

$=$

$d = 350\ mm$

5. Since $M_u < M_{ulim}$

$\Rightarrow$ Design it as singly reinforced beam.

6. $M_u = 0.36 f_{ck}\, b\, x_u\, (d - 0.42 x_u)$

$\Rightarrow 75\times10^6 = 0.36\times20\times240\, x_u \left[350 - 0.42 x_u\right]$

$\Rightarrow 75\times10^6 = \left[0.36\times20\times240\, x_u \times 350\right] - \left[0.36\times20\times240\, x_u \times 0.42 x_u\right]$

$\Rightarrow 75\times10^6 = 604800\, x_u - 725.76\, x_u^2$

$\Rightarrow 725.76\, x_u^2 - 604800\, x_u + 75\times10^6 = 0$

$\Rightarrow x_u = 151.58$ mm

$x_{umax} = 0.48d$ (for fe415)

$= 0.48\times350 = 168$ mm

Since $x_u < x_{umax} \Rightarrow$ Under-reinforced section

∴ Design is OK

7. To find Ast

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$$

$$151.58 = \frac{0.87\times415\, A_{st}}{0.36\times20\times240}$$

$\Rightarrow A_{st} = 725.50$ mm^2 ✓

$A_{stmin} = 0.85\frac{bd}{f_y} = \frac{0.85\times240\times350}{415}$

$= 172$ mm^2

$A_{stmax} = 0.04bD$

$= 0.04\times240\times400$

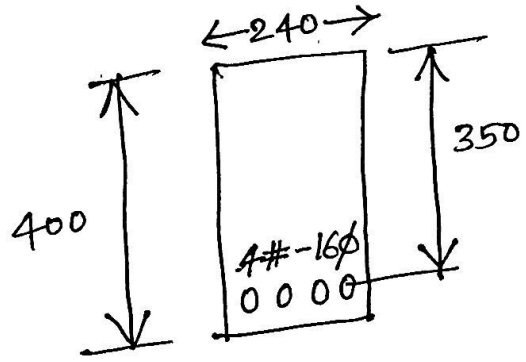
$= 3840$ mm^2

Assume 16 mm ϕ bars,

$$\text{No of bars required} = \frac{725.50}{\pi/4 \times 16^2}$$

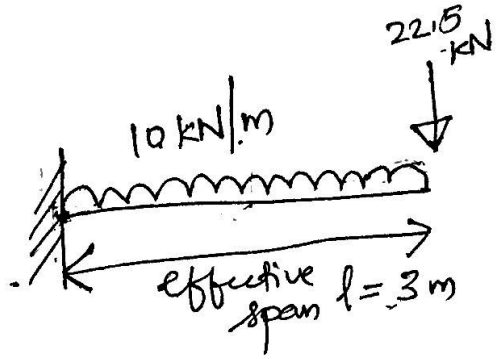
$$\approx \underline{4 \text{ bars}}$$

Provide 4 bars of 16 mm ϕ as tension reinforcement



All dimensions in mm

Q: Design a cantilever beam of effective span 3m which carries a factored UDL 10 kN/m over the entire span and a factored point load at free end. (22.5 kN) Use M20 concrete & fe.500 steel.



Ans:

$l = 3m = 3000\,mm$

1. finding d:

$$\frac{l}{d} \leq 7 \quad \text{(for cantilevers)}$$

$$\frac{3000}{d} \leq 7 \Rightarrow d \geq \frac{3000}{7}$$

$$\geq 428.57\,mm$$

Effective depth $d = 550\,mm$

Assume effective cover $= 50\,mm$

$\therefore$ Overall depth $D = 550 + 50 = 600\,mm$

2. find b

Take $b = 250\,mm$

3. finding M_u:

$W_{u\,(UDL)} = 10\,kN/m$

$W_{u\,(Point)\,@free\,end} = 22.5\,kN$

$$M_u = \frac{W_{u\,(UDL)}\,l^2}{2} + W_{u\,(point)}\,l$$

$$= \left(\frac{10 \times 3^2}{2}\right) + (22.5 \times 3)$$

$$= 112.5\,kNm$$

4. $M_{ulim} = 0.36 f_{ck} b x_{umax} (d - 0.42 x_{umax})$

$= 0.36 \times 20 \times 250 \times 0.46 \times 550 (550 - 0.42 \times 0.46 \times 550)$

$= 202 \times 10^6$ Nmm.

$= \underline{202 \text{ kNm}}$

$f_{ck} = 20$

$b = 250$ mm

$d = 550$ mm

$x_{umax} = 0.46 d$ (for Fe500)

$= 0.46 \times 550$

Since $M_u < M_{ulim}$

$\Rightarrow$ Singly reinforced beam

5. find x_u

$M_u = 0.36 f_{ck} b x_u (d - 0.42 x_u)$

$\Rightarrow 112.5 \times 10^6 = 0.36 \times 20 \times 250 x_u (550 - 0.42 x_u)$

$\Rightarrow 112.5 \times 10^6 = (0.36 \times 20 \times 250 \times 550 x_u) - (0.42 \times 0.36 \times 20 \times 250 x_u^2)$

$\Rightarrow 112.5 \times 10^6 = 990000 x_u - 756 x_u^2$

$\Rightarrow 756 x_u^2 - 990000 x_u + 112.5 \times 10^6 = 0$

$\Rightarrow \underline{x_u = 125.70 \text{ mm}}$

$x_{umax} = 0.46 d$ (for fe500)

$= 0.46 \times 550 = 253$ mm

Since $x_u < x_{umax}$

$\Rightarrow$ Under reinforced section

$\therefore$ Design is <u>OK</u>

6. Finding Ast

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$$

$$125.70 = \frac{0.87 \times 500 \times A_{st}}{0.36 \times 20 \times 250}$$

$$\Rightarrow A_{st} = 520.14 \text{ mm}^2$$ ✓

$$A_{st,min} = 0.85 \frac{bd}{f_y}$$

$$= 0.85 \times 250 \times \frac{550}{500}$$

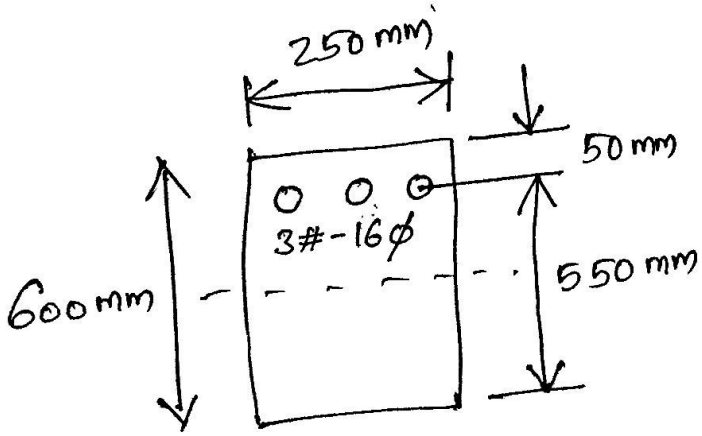
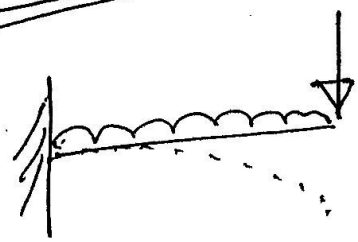
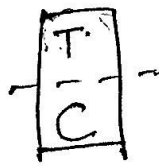
$$= 233 \text{ mm}^2$$

$$A_{st,max} = 0.04 bD$$

$$= 0.04 \times 250 \times 600$$

$$= 6000 \text{ mm}^2$$

Assume 16 mm ϕ bars,

$$\text{no: of bars required} = \frac{520.14}{\pi/4 \times 16^2}$$

$$\approx 3 \text{ bars}$$